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Comment on "Buckling in Segmented Shells of Revolution Subjected to Symmetric and Antisymmetric Loading"

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REFERENCE 1 contains an interesting study of its title subject based on an extension of previous work by Bushnell.[2] The only two numerical problems considered in the paper, viz: pure bending and symmetric lateral pressure on cylindrical shells, are well known in the literature.

It must be noted that a subsequent work by Bushnell[3] has not been quoted by the authors. In that paper, an approximate method is given for calculating the buckling load of axisymmetric structures subjected to nonuniform lateral loads. Bushnell determines the meridian having the "worst" disturbance of stress, from a linear prebuckling analysis, and then performs an axisymmetric buckling analysis, assuming this distribution to be uniform in the circumferential direction. This method is expected to yield buckling loads lower than those by a more rigorous approach, such as the one in the authors' paper.[1] Since the authors have intended to amplify Bushnell's work, a comparison of these two methods would have been appropriate and is of practical interest for people using the BOSOR4 code developed by Bushnell.[3] In their conclusions, the authors have stated that the mode of buckling under symmetric loads is either symmetric or antisymmetric, and both should, therefore, be considered to determine the lower eigenvalue. It is not clear to the present writers whether both the Fourier cosine and sine series have been considered in the buckling state variables for the numerical problems quoted and, if so, what differences have resulted.

It should also be noted that the lower limit of the harmonics considered in the Fourier series of the buckling state variables, need not be fixed at 0 or 1, as in Eq. (13), particularly for problems involving nonuniform lateral pressure, since the "effective" Fourier coefficients can start at any higher value depending on the shell geometry[4] etc. If K_L, the lower limit of the harmonic series, is fixed at 0, then K_U, the upper limit of the series, may have to be a high value for convergence; the stability determinant will then be unduly large, without significant influence on the eigenvalue that would be obtained when only the "effective" Fourier coefficients are considered. Hence, it appears to be more convenient to let $K_U = K_L + K_E$, where K_E is a small fixed number corresponding to the number of effective Fourier coefficients (minus one), and to study the variation of the eigenvalue as a function of K_L.

References 5–8 are concerned with cylindrical shells subjected to nonuniform pressures. Since these works deal with basically a similar problem, the writers have listed them here, as they may be of some interest to the workers in this subject. In all these works, the stability criterion used is based on the second variation of the energy that is expressed in terms of prebuckling strains and the virtual displacement components. The prebuckling analysis is carried out using a linear theory. In Ref. 5, only simply supported shells are considered, hence a Fourier series representation in the axial co-ordinate is made as in Ref. 4. In Refs. 6–8, the writers have used algebraic polynomials in the axial coordinate for the virtual displacement components and obtained good results for various combinations of different boundary conditions at the circular ends. Supporting experimental evidence is contained in Ref. 8.

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